- ANALISI II -

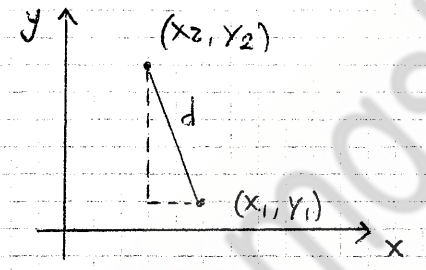
29/02/16

$\mathbb{R}^n$ SPAZIO VETTORIALE

$\vec{x} \in \mathbb{R}^2$, (x, y)

$\vec{x} \in \mathbb{R}^3$, (x, y, z)

- DISTANZA TRA VETTORI IN $\mathbb{R}^n$ -



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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\vec{x} = (x_1, x_2 \dots x_n)$

$\vec{y} = (y_1, y_2 \dots y_n)$

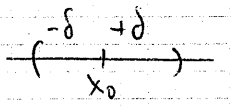
$$d(\vec{x}, \vec{y}) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_n - y_n)^2}$$

Esempio

Calcolare la distanza tra $(-1, 3, -7)$ e $(0, -\pi, \sqrt{2})$

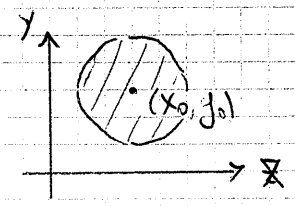
$$d = \sqrt{(-1)^2 + (3 + \pi)^2 + (7 + \sqrt{2})^2}$$

- INTERVALLI -

In analisi 1; 

$$I(x_0, \delta) = \{ x \in \mathbb{R} \mid |x - x_0| < \delta \}$$

In analisi 2;

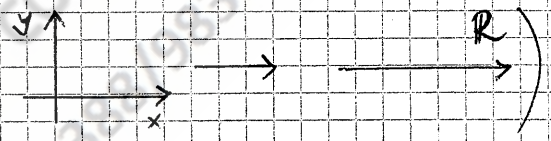
$$\{(x,y) \in \mathbb{R}^2 \mid d < \delta\} \quad \text{}$$

$$\| \\ B_\delta(x_0, y_0)$$

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Def Una funzione di n variabili è una legge che associa a vettori in $\mathbb{R}^n$ un numero reale

$$\Omega \subseteq \mathbb{R}^n \ni \vec{y} \longrightarrow f(\vec{y}) \in \mathbb{R}$$

Esempio - $f(x,y) = x^2 + \operatorname{Sen} y$ 

$$\mathbb{R}^2$$

$f(0,0) = 0 \qquad D = \mathbb{R}$

- $f(x,y) = \ln|xy|$

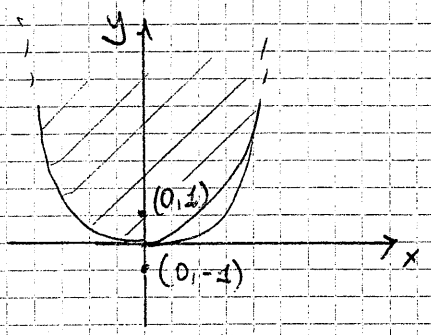
$D = \{(x,y) \in \mathbb{R}^2 \mid (x \cdot y) \neq 0\}$ TUTTI I PUNTI ESCLUSO GLI ASSI

$$\| \\ \{(x,y) \in \mathbb{R}^2\} \setminus (\{x,0\} \cup \{0,y\})$$

Esempio Rappresentare il dominio di $f(x,y) = \dfrac{\sqrt{x^2-y}}{\ln(x+y)}$

$$D(f) = \left\{(x,y)\in\mathbb{R}^2 \;\middle|\; x^2-y\geq 0,\quad x+y>0,\quad x+y\neq 1\right\}$$

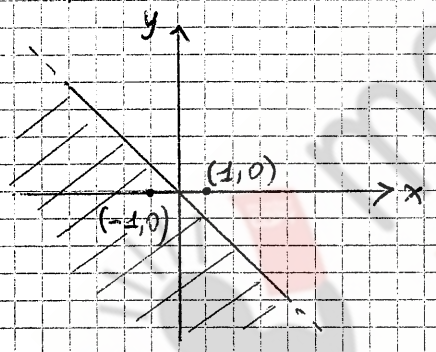
I $\left\{(x,y)\in\mathbb{R}^2 \;\middle|\; x^2-y\geq 0\right\}$



$y \geq x^2$

SOSTITUISCO DUE PUNTI DI PROVA E CAPISCO LA REGIONE DI SPAZIO CHE SODDISFA L'EQUAZIONE

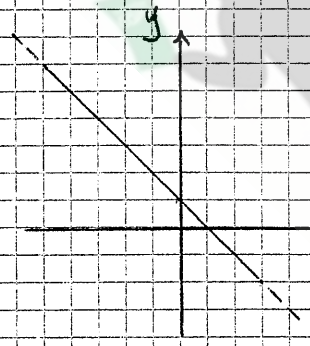
II



$y \geq -x$

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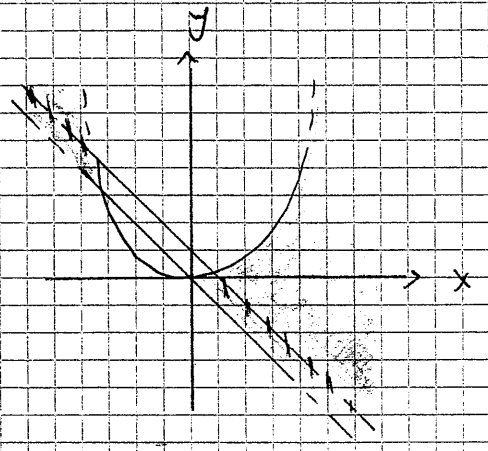
III



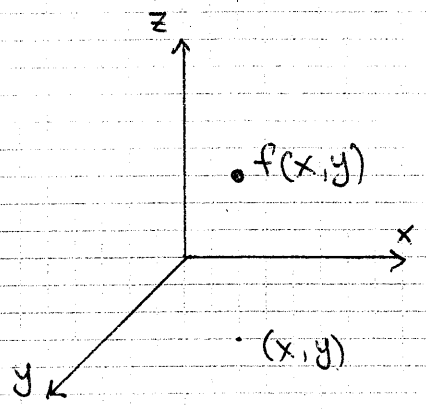
$y \neq -x+1$

TUTTI I PUNTI TRANNE LA RETTA

INTERSEZIONE



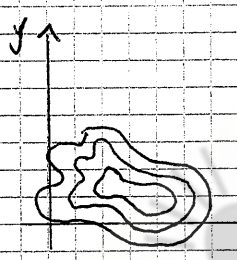
- GRAFICI DI FUNZIONI -



$$G = \{(x,y) \in \mathbb{R}; \ f(x,y)\}$$

Superficie in $\mathbb{R}^3$

- CURVE DI LIVELLO -



$$\{(x,y) \in \Omega \in \mathbb{R}^2 \mid f(x,y) = \lambda\}$$

Sottoinsiemi in $\mathbb{R}^2$ di un grafico che avrebbe la necessità di vivere in $\mathbb{R}^3$

Esempio

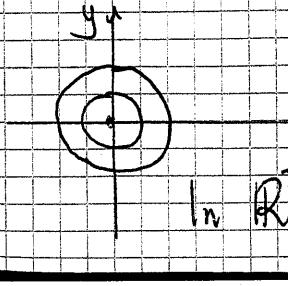
$f(x,y) = x^2 + y^2$ $\qquad C_\lambda = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = \lambda\}$

$D(f) = \mathbb{R}^2$

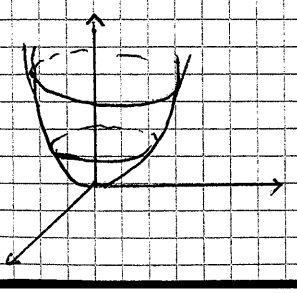
Se $\lambda < 0$, $C_\lambda = \emptyset$

Se $\lambda = 0$, $C_0 = \{(0,0)\}$

Se $\lambda > 0$, C_λ = circonferenza di centro $(0,0)$ e $r = \sqrt{\lambda}$



In $\mathbb{R}^2$



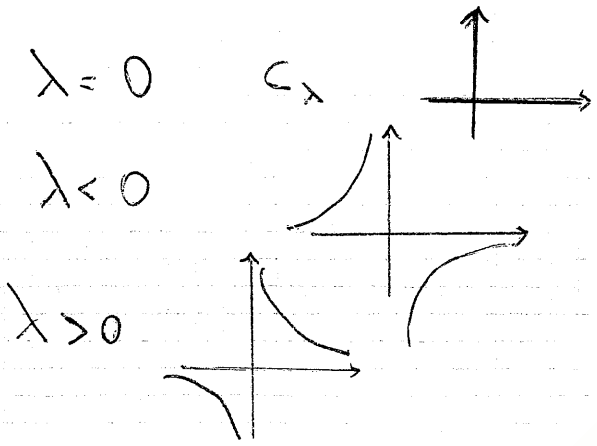
In $\mathbb{R}^3$

Esempio $f(x,y) = xy$ $\quad C_\lambda = \{(x,y) \in \mathbb{R}^2 \mid xy = \lambda\}$

$\lambda = 0$ $\quad C_\lambda$

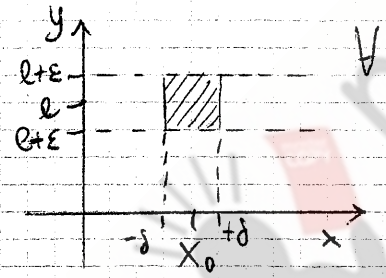
$\lambda < 0$

$\lambda > 0$



- LIMITI -

In Analisi 1: $\lim_{x \to x_0} f(x) = \ell$ $\quad \ell \begin{cases} +\infty \\ -\infty \\ \ell \in \mathbb{R} \\ \text{NON ESISTE} \end{cases}$ $\quad f(x)$ $x \in \Omega \subseteq \mathbb{R}$



$\forall \varepsilon > 0 \ \exists \ \delta > 0$ t.c. $|f(x) - \ell| < \varepsilon \ \ \forall x \in I_\delta(x_0) / (x_0)$

In Analisi 2: $f(x)$ $\quad \vec{x} \in \Omega \subseteq \mathbb{R}^2$

$\lim_{\vec{x} \to \vec{x}_0} f(x) = \begin{cases} +\infty \ (1) \\ -\infty \ (2) \\ \ell \in \mathbb{R} \ (3) \\ \text{N.E.} \ (4) \end{cases}$

(1) $\forall M > 0 \ \exists \ \delta > 0$ t.c. $f(x,y) > M$ se $(x,y) \in B_\delta(x_0, y_0) / (x_0, y_0)$

